# HYDRODYNAMICS OF FLOW IN A PERMEABLE CHANNEL WITH INJECTION FROM TWO SIDES

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UDC 532.526

The main peculiarity of flow in a channel with a transverse mass flux in comparison with external flow over a plate with injection is that large accelerations develop in the channel owing to the continuous mass supply.

The most extensive investigations of established turbulent flow in a round pipe with uniform injection along the length were made in [1]. In that report the results of the measurement of the velocity and pressure distribution are given for two cases: Developed turbulent flow exists at the entrance to the permeable section of the pipe; a zero velocity at the entrance.

It was shown in [1] that with developed turbulent flow at the entrance to a permeable pipe the decrease in the coefficients of friction agrees with the decrease in the coefficients of friction for external flow for the investigated range of injection rates (the ratio $\rho_w v_w/\rho_0 \bar{u}$ was varied from 0.00246 to 0.0584, where $\rho_w$ and $v_w$ are the density and velocity of the injected liquid and $\rho_0$ and $\bar{u}$ are the density and the viscosity, averaged over the cross section, at the entrance to the permeable part of the pipe).

In the present article we give the results of an experimental and theoretical investigation of the flow in a rectangular channel of great length ($l/h \sim 30$, where $l$ is the length of the permeable section and h is the height) when there is developed turbulent flow at the entrance to the porous section.

## 1. Description of Experimental Installation

The working section is a channel with a rectangular cross section of h × 30 mm (h = 5.8-4.4 mm) with transparent side walls. The upper and lower walls were formed by two mounts with porous plates 24 × 200 mm in size cemented into them. The plates were made from L-3 polymethyl methacrylate powder by sintering. An investigation of the porosity of the surface under a microscope showed that the average height of the projections, consisting of sandy roughness, was 0.02 mm. The injection rates through the two walls (water was injected into water) were regulated independently of each other. Preliminary measurements showed that the pressure drop at a porous plate is about an order of magnitude greater than the longitudinal pressure drop in the channel. Thanks to this, $v_w$ was constant over the entire length of the permeable section.

To obtain developed turbulent flow at the entrance to the experimental section of the channel there was an impermeable section (a channel with the same through cross section) 750 mm long. The distribution of velocities and their pulsations were measured by the method of stroboscopic visualization of the stream with marker particles of aluminum powder 5-20 μm in size. A diagram of the measurements is shown in Fig. 1, where 1 is the flash lamp, 2 and 3 are the slit and objective for obtaining the light knife edge, 4 is the camera, the objective lens of which was focused on the central plane of the channel, 5 is the working section, and 6 are the transparent side walls. The depth of field of the camera objective lens was no more than 0.5 mm. By this method we measured the distributions of velocities and pulsations over the channel cross section at different distances (cross sections $x_1$, $x_2$, etc.) from the start of the permeable section for three relative injection rates, $v_w/u_m = 3\cdot10^{-3}$, $6\cdot10^{-3}$, and $1\cdot10^{-2}$ ($u_m$ is the local value of the maximum of the longitudinal velocity component). The exact value of this parameter is given in Table 1. The distribution of static pressure along the channel length was measured with the help of six openings 0.5 mm in diameter, uniformly distributed in a side wall.

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Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 62-67, November-December, 1981. Original article submitted November 6, 1980.

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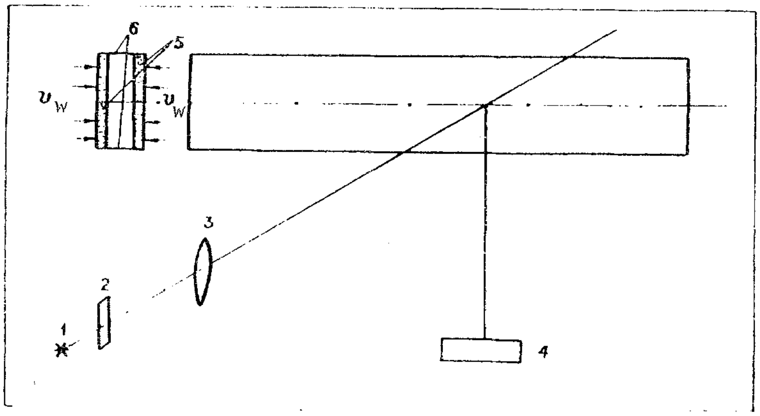


Fig. 1

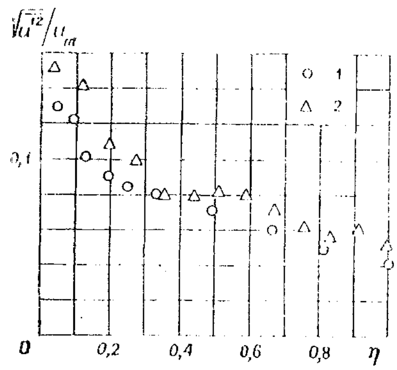


Fig. 2

TABLE 1

| $v_w$, m/sec | x, mm | $v_w/u_m$ |
|---|---|---|
| 0,047 | 50 | 0,0129 |
| | 98 | 0,0104 |
| | 158 | 0,0095 |
| 0,0104 | 28 | 0,0072 |
| | 78 | 0,0069 |
| | 180 | 0,0058 |
| 0,0052 | 28 | 0,0033 |
| | 98 | 0,0032 |
| | 179 | 0,0030 |

## 2. Experimental Results

The control measurements included the following.

1. Measurement of the distribution of root-mean-square pulsations of the longitudinal velocity component at the entrance to the working section with porous walls in the absence of injection (Fig. 2, points 1 are Reichardt's data [2], 2 are the authors' data, $\eta = 2y/h$, and y is the distance to the porous wall).

2. Measurement of the pressure drop along the length of the channel (Fig. 3, points 1, x is the distance from the entrance of the channel with permeable walls, the dashed line corresponds to a calculation by the Blasius equation for smooth pipes using the effective diameter [2]). The data presented in Figs. 2 and 3 show that the flow was fully developed turbulent flow at the entrance to the channel with porous walls.

3. A test of the two-dimensionality of the flow. For this purpose we measured the velocity distributions with the maximum investigated value of the injection rate at the channel axis and at a distance of 3 mm to one side of it. The measurements showed that the investigated flow is two-dimensional.

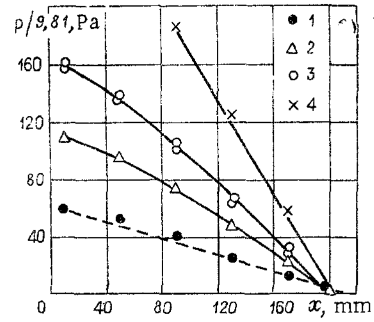

Fig. 3

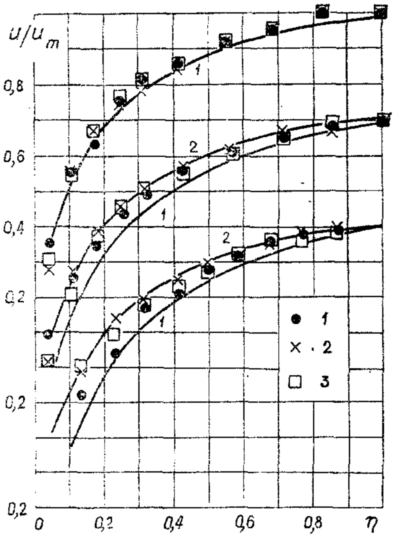

Fig. 4

The velocity distributions corresponding to the above-indicated injection rates are presented in Fig. 4. Curve 1 is the calculated velocity distribution obtained for a plate with injection [3],

$$\omega = (1 - \omega_0)\sqrt{\Psi + b} + (1 - \omega_0)^2 b/4,$$

$$\text{where } \omega = u/u_m;\ b = \frac{2}{c_{f_0}}\frac{v_w}{u_m};$$

$\Psi = \left(\frac{c_f}{c_{f_0}}\right)_{\mathrm{Re}^{**}}$ is the ratio of coefficients of friction taken at the same value of the Reynolds number Re** for the given conditions and standard conditions (flow on a plate), which was calculated from the formula

$$\Psi = (1 - b/b_{cr})^2,\ b_{cr} = 4(1 + 0.83/\mathrm{Re}^{**0.14}),$$

where $\mathrm{Re}^{**} = u_m\delta^{**}/\nu$; $\delta^{**}$ is the thickness of momentum loss; $\nu$ is the coefficient of kinematic viscosity; $c_{f_0}$ was calculated from the Kármán equation and the local value of Re**; $\omega_0$ is the velocity distribution at the entrance to the porous section of the channel. In calculating $b_{cr}$ from the data of [4] a correction was introduced for the influence of acceleration. The series of points grouped about curve 1 of Fig. 4 corresponds to an injection rate $v_w/u_m = 0.003$; 2) 0.006; 3) 0.01. The measurements were made in the following cross sections: $v_w/u_m = 0.003$, points 1) x = 28 mm, 2) x = 98 mm, 3) x = 179 mm; $v_w/u_m = 0.006$, points 1) x = 28 mm, 2) x = 78 mm, 3) x = 180 mm; $v_w/u_m = 0.01$, points 1) x = 50 mm, 2) x = 98 mm, 3) x = 158 mm.

It is seen from Fig. 4 that with an increase in the relative injection rate, starting with about $v_w/u_m = 0.006$, the experimental velocity distribution departs more and more from the calculated one.

In this connection, it was assumed that these departures are caused by the influence of accelerations developing in the channel. To calculate the magnitude of this influence we used assumptions which have proven valid in calculations of the joint influence of many factors using the limiting theory of [3], to wit,

$$\Psi = \Psi_{\Lambda,b} = \Psi_\Lambda \Psi_b,$$

$$\omega(\Lambda, b) = 1 - (1 - \omega_0(\Lambda) \sqrt{\Psi_b + b} + (1 - \omega_0(\Lambda))^2 \frac{b}{4},$$

where $\Psi_b = (1 - b/b_{cr}(\Lambda))^2$; $\Lambda = (h/2\tau_w)dp/dx$; $\tau_w$ is the shear stress at the wall; $b = (v_w/u_m)(2/c_f(\Lambda))$, while

$$\sqrt{\Psi_\Lambda} = \int_0^1 \sqrt{\frac{\tilde{\tau}_0}{\tilde{\tau}(\Lambda)}}\, d\omega_0(\Lambda); \tag{1}$$

$$\sqrt{\Psi_\Lambda}(1-\omega_0(0)) = \int_0^\omega \sqrt{\frac{\tilde{\tau}_0}{\tilde{\tau}(\Lambda)}}\, d\omega_0(\Lambda), \tag{2}$$

$\tilde{\tau} = \tau/\tau_w$ is the shear stress distribution; $\omega_0(0)$ is the velocity distribution for $\Lambda = 0$.

In order to use Eqs. (1) and (2) one must know the shear stress distribution $\tau(\Lambda, \eta)$. For established flow in an impermeable pipe we have

$$\tilde{\tau} = 1 + \Lambda\eta \quad (\eta = 0 - 1). \tag{3}$$

The use of this approximation for $\Lambda < -1$ gives $\tilde{\tau} < 0$. Estimates of $\Lambda$ for the investigated flows showed that $\Lambda < -1$. An approximating function for $\tilde{\tau}$ suitable for $\Lambda < -1$ was obtained on the basis of the following considerations. For gradient flow on a plate with small $\eta$ the distribution of $\tilde{\tau}$ coincides with the corresponding distribution for flow in a channel. In this connection, it was assumed that the approximation

$$\tilde{\tau} = \tilde{\tau}_0/[1 - (2 + \Lambda)\eta(1 + 2\eta)] \tag{4}$$

from [5], which well describes the experimental data for gradient flow on a plate for $\Lambda < -2$, will give the distribution of $\tilde{\tau}$ for small $\eta$ being sought. Neglecting terms of order $\eta^2$ in comparison with terms of order $\eta$ in (4), we obtain

$$\tilde{\tau} = \tilde{\tau}_0/(1 - \Lambda\eta). \tag{5}$$

In using this expression (5) for flow in a channel with injection, we assumed that $\tilde{\tau} = 1 + \Lambda_0\eta$, $\Lambda_0$ being the pressure-gradient parameter for an impermeable channel while $\Lambda$ is the parameter corresponding to the additional pressure drops arising because of the continuous mass supply. It is seen that with $\Lambda = 0$ Eq. (5) changes into (3).

It is interesting to note that the approximation of $\tilde{\tau}$ obtained in this way satisfies the same boundary data for gradient flow on a plate and, in contrast to (4), can be used at all negative values of $\Lambda$, including the region of $\Lambda > -2$.

In Fig. 5 the data of [6] for $\Lambda = -4.25$ are compared with the dependences (4) and (5) (curves 2 and 3, respectively). Curve 1 is plotted from (5) for $\Lambda = 0$.

Equations (1) and (2) can be rewritten in the form

$$\int_0^1 \sqrt{1 - \Lambda\eta}\, d\omega_0(\Lambda) = \sqrt{\Psi_\Lambda}; \tag{1a}$$

$$\int_\omega^1 \sqrt{1 - \Lambda\eta}\, d\omega_0(\Lambda) = \sqrt{\Psi_\Lambda}\,(1 - \omega_0(0)), \tag{2a}$$

where $\omega_0(\Lambda)$ and $\Psi_\Lambda$ are unknown functions. The system of equations (1a) and (2a) allows us to find $\omega_0(\Lambda) = f(\Lambda, \eta)$ and $\Psi_\Lambda$ when dp/dx, h, and $\omega_0(0)$ are given.

The originally proposed method was used in order to ascertain that the influence of acceleration on the quantity $\Psi$ and the velocity distribution can be significant. For the calculation we used the pressure drop along the length of the channel measured in the ex-

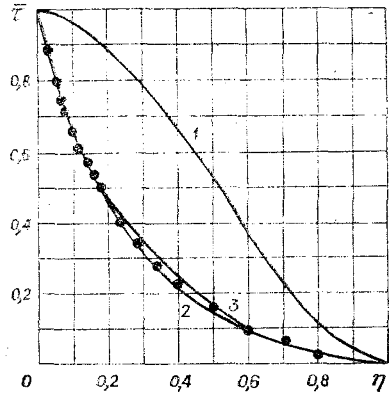


Fig. 5

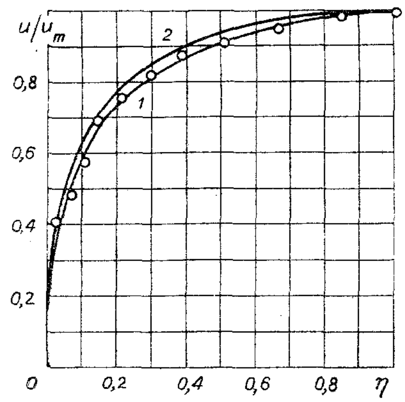


Fig. 6

periments with injection [Fig. 3, points 1) $v_w/u_m = 0$; 2) $v_w/u_m = 0.003$; 3) $v_w/u_m = 0.006$; 4) $v_w/u_m = 0.01$]. The system of equations (1a) and (2a) was solved by the method of successive approximations. In the zeroth approximation we took $\omega_0(\Lambda, \eta) = \omega_0(0, \eta)$ and $\tau_w = c_{f_0}\rho u_m^2/2$.

For the pressure gradient with $v_w/u_m = 0.01$ and x = 158 mm, $\Psi = 1.3$, while the deformation of the velocity profile is represented by curve 2 in Fig. 6 and curve 1 approximates the authors' experimental data for $\Lambda \approx 0$ in the channel without injection.

The results of the calculations of flow in a channel with allowance for injection and the pressure gradients which arise are represented by curve 2 in Fig. 4.

The rather considerable change (30%) in the value of $\Psi_\Lambda$ under the influence of a favorable pressure gradient should be noted. This result does not, however, contradict the assumption that the friction law is conservative for the entire range of accelerations up to relaminarization (for flows without injection), since injection considerably widens the range of pressure gradients in which turbulent flow still occurs (cf. [7]). Thus, for the above-described experiments with $v_w/u_m = 0.01$, 0.006, and 0.003 the value of the parameter K was $\sim 3.9 \cdot 10^{-6}$, $2.8 \cdot 10^{-6}$, and $1.5 \cdot 10^{-6}$, respectively.

As mentioned above, the pressure gradient measured in the experiments was used in the calculations. In concrete calculations this value is unknown. In this connection, let us consider established two-dimensional flow in a permeable channel with uniform injection along the length. Since $\int_0^1 \omega^2 d\eta \approx \left(\int_0^1 \omega d\eta\right)^2$ in a wide range of n (from 1/2 to 0, n being the exponent of the approximation $\omega = \eta^n$), from the momentum equation integrated over the height of the channel we obtain

$$-dp/dx = 2\tau_w/h + \rho\bar{u}d\bar{u}/dx,$$

where $\bar{u} = u_0 + u_w 2x/h$; $u_0$ is the velocity at the entrance to the permeable section, averaged over the cross section. To calculate the first term of this expression for the entire range of injection rates we can set $\Psi = \Psi(b)$. Actually, for large values of b the contribution of the first term becomes insignificant and the errors in determining $\tau_w$ from $\Psi = \Psi(b)$ reflect little on the accuracy of calculations of the developing pressure gradient. For small b, when the contribution of the first term grows, the accelerations which develop are small and one can take $\Psi = \Psi(b)$.

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# INVESTIGATION OF THE PULSATION CHARACTERISTICS OF AN ASCENDING GAS—LIQUID FLOW


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UDC 532.529.5


A great deal of attention has been paid recently to the study of the local structure of two-phase gas—liquid flows. This is related to the diversity of the flow modes and the large quantity of parameters and governing characteristics of two-phase flows. Consequently, generalization of the average total parameters (friction losses, heat elimination coefficient) turns out to be difficult. Investigations performed at this time of the local gas—liquid flow characteristics averaged with respect to time [1, 2] must be supplemented by a more detailed investigation of the pulsating flow characteristics to clarify the interaction mechanism between the liquid and gas phase turbulence.

The local gas—liquid flow characteristics averaged with respect to time were investigated in a vertical pipe in [3, 4]. The purpose of this paper is to investigate certain pulsating characteristics of an ascending two-phase flow.

The experiments were performed on the apparatus from [3]. A vertical pipe with 15-mm inner diameter and 5-m length was the working section. A two-phase flow was organized by inserting gas bubbles of identical size in the fluid by using the unit from [4].

Measurements were executed by using an electrochemical method [5, 6]. An electrolyte containing 0.5 N caustic soda and 0.005 N ferri- and ferrocyanide in distilled water was the working fluid. The fluid was kept in a thermostat at 20 ± 0.2°C to maintain the physical properties constant.

A velocity transducer of "frontal point" type of 0.05-mm diameter was used to determine the mean fluid velocity and the fluctuations of its longitudinal component. The transducer


Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 67–71, November-December, 1981. Original article submitted December 17, 1980.




0021-8944/81/2206-0792$07.50